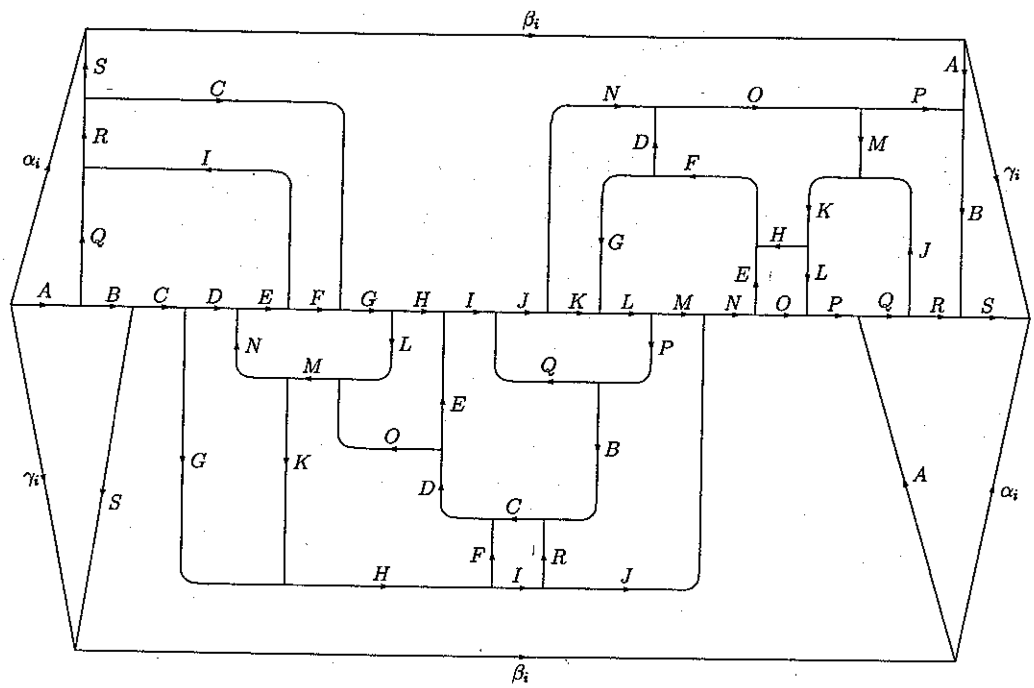

Figure 27: Diagram $*i$

have the following equation *in* $\pi_1(T_i)$:

$$
\begin{aligned}
[\varphi_i(m_w)] &= \varphi_i^{\#}([m_w]) \\
&= \varphi_i^{\#}(D_{k+1,k+1}[x_w] - D_{k,k+1}[y_w]) \\
&= D_{k+1,k+1}\,\varphi_i^{\#}([x_w]) - D_{k,k+1}\,\varphi_i^{\#}([y_w]) \\
&= D_{k+1,k+1}[\alpha_i\,\beta_i] - D_{k,k+1}[\beta_i\,\gamma_i] \\
&= D_{k+1,k+1}[\alpha_i\,\beta_i] - D_{k,k+1}[\beta_i\,\alpha_i\,\alpha_i^{-1}\,\gamma_i] \\
&= D_{k+1,k+1}[\alpha_i\,\beta_i] - D_{k,k+1}([\beta_i\,\alpha_i] + [\gamma_i\,\alpha_i^{-1}]) \\
&= (D_{k+1,k+1} - D_{k,k+1})[\alpha_i\,\beta_i] - D_{k,k+1}[\gamma_i\,\alpha_i^{-1}]
\end{aligned}
$$

Two loops $\alpha_i\,\beta_i$ and $\gamma_i\,\alpha_i^{-1}$ are embedded in T_i such that $\alpha_i\,\beta_i = T_i \cap H_i'$ and $\alpha_i\,\beta_i$ intersects $\gamma_i\,\alpha_i^{-1}$ at a point. By Lemma 3.3(i), we get $D_{k+1,k+1} - D_{k,k+1} = p_i$. So the fiber type is $(p_i, -D_{k,k+1})$.

(b) The case k is even

The proof is similar to the case (a). By Proposition 5.6, we have the following conditions in $\pi_1(q_i/p_i)$.

$$[\varphi_i^{-1}(\gamma_i\,\alpha_i^{-1})] = p_i\,[l_w] + q_i\,[m_w].$$

This means that the core of $T(q_i/p_i)$ is (p_i, q_i) type singular fiber. Also, we get